Chapter 2:
Full Belief and The Pursuit of Certainty

"Doubt is the chastity of the mind." R. Zelazny.
"Doubt is not a pleasant mental state but certainly is a ridiculous one." Voltaire.

2.1 Ignorance: The Views from Dogmatism and Skepticism

Without attempting a thoroughgoing survey of classical philosophy, some initial insights may be gained into Western intellectuals' traditional orientations toward ignorance by briefly examining the varieties of ignorance referred to in classical philosophical literature. Every theory of knowledge draws a distinction between knowledge and ignorance, and most between ignorance in the sense of incomplete knowledge and ignorance in the sense of erroneous belief. The earliest forms of philosophy amount to various kinds of dogmatism or skepticism, both of which necessarily refer to and utilize ignorance. I will discuss dogmatism first.

Dogmatic philosophers such as Plato or Locke require absolute certainty about knowledge. In fact, dogmatists equate the two; anything we know we are certain of and vice versa. To a complete dogmatist, everything is known that can or must be known, at least by some ultimate authority. Traditionally this authority is omniscient, although it is possible for the dogmatist to countenance specialists as authorities. Any beliefs or assertions differing from or truncating dogma constitute heresies, and it is noteworthy that the most dogmatic of religions (e.g., many versions of Christianity) attach moral culpability as well as an attribution of error to heresy. To err is not merely human, it is also bad. Dogmatists distinguish doubt from erroneous belief, where doubt usually means to withhold assent both from a proposition and its negation. But doubt also traditionally carries a moral stigma, since the basis for many dogmas rests on faith (Luther, for instance, was adamant that one could not be a Christian and a skeptic simultaneously).

Dogmatists do permit two kinds of ignorance that are not necessarily morally reprehensible. One is conscious ignorance, if accompanied by the appropriate deference to knowledgeable authority. Indeed in many Western and nonWestern knowledge systems, certain kinds of knowledge are not accessible or permitted under various circumstances. The old term 'mystery' at one time referred to both the unknown and the secret. If everything is known, then a mystery can only exist for one who has not been initiated into the inner circle of those who know. Some matters may be taboo, in which case it is knowledge rather than evil which is wicked. So, for the dogmatist, a natural counterpart to conscious ignorance (mystery) is taboo, and in some systems any kind of irrelevant matter (e.g., fantasy or fiction as viewed by Puritans) is considered taboo.

The second kind of morally excusable ignorance is innocence, which usually denotes an unconscious ignorance that nonetheless is free of heresy. This is not to say that all meta-ignorance is excusable to dogmatists; that point is clearly articulated by modern legal codes which deny ignorance of the law as an excuse. Innocence, in some dogmatic systems, is exalted above fire-tested faith and belief, which then leads to a complicated and paradoxical arrangement whereby the innocent must be protected by knowledgeable but tainted guardians.

Ignorance, then, tends to be construed by dogmatists according to the culpability of the ignoramus. Aetheists, heretics, and agnostics all are reproachable, even though the first two believe erroneously while the latter merely doubts or suspends judgment. Innocents and those unitiated into the supreme mysteries escape censure by the dogmatist, but only if they keep their places.

Modern dogmatism takes a somewhat mitigated form, although mitigated dogmatism itself is also an ancient style of philosophy. Most modern dogmatists hold only a crucial subset of their beliefs as dogma (e.g., dogmatic rationalists), thereby permitting doubt or critical inquiry into their nonessential beliefs. Not everything is known, and perhaps there is no omniscient being. Indeed, some dogmatists claim the best defense against skeptics is to retain the smallest and most self-evident set of dogmas possible. However, moral sanctions still apply to those in error or doubt regarding the central dogmas, even if those sanctions no longer include being burned at the stake.

Furthermore, while the modern dogmatist may hold that whatever is certain also is known, she or he usually does not require the converse. Mitigated dogmatism admits uncertainty into the realm of knowledge, albeit under careful control. Usually this is done by creating a category for uncertain knowledge or at least knowledge that is not demonstrable, as when Socrates drew his famous distinction between knowledge and opinion. While not denying that one could have correct opinions, philosophers following this line claim that opinion must be accompanied by some degree of probability or doubt.

Now, let us consider how ignorance is viewed by the opposite to radical dogmatism: Pyrrhonist (or complete) skepticism. Complete skepticism maintains that nothing can be certain, nor can anything be known. No one is justified or reasonable in their assertions about reality. Like its dogmatic counterpart, total skepticism is unarguable because any argument requires the disputants to agree on some criterion for determining whose argument is correct, and both dogmatists and skeptics accept no terms other than their own. Those who hope to refute the skeptics (or their modern counterparts, the radical relativists) by pointing out that skepticism itself is unjustified according to its own tenets provoke the rejoinder that skeptics do not see apparent inconsistency as a difficulty since they do not believe in logic anyway.

The complete skeptic therefore has a monolithic view of ignorance which is all-embracing. There is no such thing as error, irrelevance, or innocence. Doubt is universal and overwhelming. All is utter mystery and no one can say what is erroneous to believe. Nor is the complete skeptic attracted to the concept of degrees of uncertainty. As Montaigne pointed out, to acknowledge one proposition to be more likely than another is to incline to one rather than the other, which is what graded measures of uncertainty (e.g., probability) require us to do. Radical skeptics will have none of it.

Restricted or mitigated skepticism, on the other hand, does involve distinctions among different kinds of ignorance. Usually the mitigated skeptic allows at least some comparable assessment of propositions such that one is decidably better than the other. Humean skepticism, for instance, requires only that we find the most probable opinion with a degree of certainty sufficient for action. A modern example is Popper's falsificationism, which holds that scientific hypotheses may not be proved but they can be confirmed in degree or outright disconfirmed. Once this is permitted then most of the classical distinctions among various kinds of ignorance become discussable, specifically doubt, error, and conscious ignorance.

This observation brings us to an interesting point. Ignorance is uncomplicated only for those philosophical positions that come close to embracing total dogmatism or complete skepticism. Moderated skepticism and dogmatism both entail more complex descriptions of ignorance. Why should this be the case? The primary reason appears to be the enhancement of strategic control over ignorance. A philosopher who places limits on human knowledge or ignorance must then classify, specify, and describe those limits or else hedge bets on where the limits are. Opinion, doubt, and probability are therefore natural candidates for a buffer zone between that which is certainly true and that which is certainly false.

Regardless of their stance on ignorance and knowledge, traditional Western philosophies universally value knowledge and seldom esteem ignorance. Even radical skeptics do not adopt a moralistic counterpart to the dogmatist's stance on knowledge and ignorance. The closest Western philosophers come to valuing ignorance is either the glorification of specific kinds on moral grounds, as a necessary precursor to gaining knowledge, or as a necessary condition for freedom of action. Thus Rousseau exalts the innocence of the savage but does not contradict the proposition that knowledge itself is beneficial. Awareness of one's own ignorance is widely considered a hallmark of intellectual sobriety, but only as an indication that one is prepared to learn or discover new knowledge. Skepticism itself originated from the Greek term for inquirer and usually plays a gadfly role in philosophical discourse.

The claim that complete knowledge of the universe entails determinism, thereby robbing people of freedom of action, is perhaps the highest valuation of ignorance in Western intellectual traditions. It is one that we shall encounter on several occasions when considering modern accounts of ignorance (especially uncertainty). Freedom, after all, is a positively valued version of uncertainty in conventional terms. Interestingly, chance itself has been argued for on these grounds. The pragmatist James, for example, defined chance as the negation of necessity which in turn creates the opportunities for human free will to exercise itself. Erich Fromm (1947) went even further and declared that the quest for certainty not only restricts freedom but blocks the search for meaning. Only uncertainty impels humanity to develop its full powers.

Now let us return to mitigated dogmatism and skepticism and the treatment of ignorance. Once the position is taken that some matters are certain but others may not be, one question that immediately arises is why people do not know everything in the first place. The Sophists wrestled with what they regarded as a paradox about the existence of erroneous beliefs. If a person believes that which is not, then does that person believe anything at all? If this is the only possible world, then how is ignorance in the sense of belief in nonentities possible? Plato's correspondence theory of truth and error easily answered this question but begged another, namely how erroneous belief arises in the first instance.

Many philosophers (e.g., Descartes), attempting to reconcile human error with the concept of an omniscient and all-loving God, asked how people could possess false beliefs when God not only knows they are false but wishes those same people well. The other side of this coin would have us consider a cosmic deceiver, Descartes' 'evil genius', who deliberately leads human perceptions astray. Descartes' famous and remarkable refutation of these difficulties is that all error in human thought is the product of human volition. This is not to say that Descartes claimed that people deliberately adopt false ideas, but rather that they are capable of assenting to propositions whose validity has not been properly established. This answer to the question of how error arises tosses the ball into the psychologists' court, which we will visit in Chapter 5.

More recent philosophical interests in ignorance have shifted from their traditional moorings in two important respects. One is a renewed interest in what might be generically termed 'nonsense', while the other is an increased moral emphasis on justification of belief (rationality or sanity). Concerns with nonsense were initially raised by the logical positivist theory of verifiability which banished all metaphysical statements as meaningless. Philosophers suddenly had to take stock of whether their own utterances amounted to nonsense. Verifiability theory posed insurmountable problems, and the criteria for nonsense shifted to linguistic analysis, with which philosophers attempted to draw a boundary between the merely false and the outright nonsensical.

The upsurge in rationalist-irrationalist debates during the same period points to a shift in the locus of moral concerns about ignorance. Although rationalisms of various kinds have been proposed since antiquity, it is a relatively modern set of mores which holds that the measure of sanity is not the specific contents of one's belief system, but rather one's way of thinking. Although we still find the older style of heretic hunting in orthodoxy-ridden societies such as the Soviet Union (where anti-Marxist dissidents or the openly religious have been labelled 'schizophrenic' and incarcerated in mental institutions), the modern mental health movements in psychology and psychiatry reflect a corresponding shift in philosophy away from identifying heresy with wrong beliefs and toward diagnosing insanity or irrationality on the basis of wrong thinking. In short, the locus of moral culpability has moved to epistemological ignorance.

Like insanity, irrationality is not considered a philosophical sin, nor do philosophers believe that vice follows from ignorance and virtue follows from knowledge. However, more than one modern philosopher has used sickness and mental illness as a metaphor for improper epistemology or lack of certitude (cf. Wittgenstein 1958: 91, 155; and 1983: 132). The image of Bertrand Russell grasping after certainty as worshippers clutch at faith is supplanted by ignorance as a mental disease and philosophers as therapists who enable themselves and others to "see the world aright." Their clients are the subrational laypeople whose commonsense biases and foibles mislead them into illusions. In this sense, modern attempts to provide secure foundations for knowledge may be viewed as mental health campaigns, and the campaign strategies used are a rich source of information about how Western culture deals with ignorance.

Most thinkers who work with uncertainty or other varieties of ignorance are not philosophers. Often they are scientists or professionals who are trained and motivated to adopt a collection of attitudes and beliefs about ignorance which might well be termed a moderate form of dogmatism. There is a central set of axioms, assumptions, beliefs, or credos which are beyond critical inquiry. These are extended by a combination of empiricist and logical guidelines to form a body of professional knowledge which is revisable but conservative. Like many philosophers, these thinkers and doers distinguish between error and incompleteness, and they have irrelevancy rules as well. Because they are not philosophers, however, they tend to judge the worth of knowledge or ignorance according to extrinsic criteria, and their resemblance to pragmatists is striking. I shall call their orientation toward ignorance 'normative pragmatism'.

The remainder of this chapter, then, consists of a limited survey of normative pragmatists' practices in handling ignorance, followed by an historical sketch of a well-known philosophical 'mental health campaign' in mathematics. These two sections will provide crucial examples, insights, and clues for an understanding of modern trends in the management of ignorance.

2.2 Examples of Traditional Normative Pragmatism: Structural Engineering and Jurisprudence

Most professions in modern society provide services under conditions of less than complete information or knowledge. Medical practitioners, social workers, lawyers, engineers, teachers, and clinical psychologists all risk failure to some extent in their respective practices. Failure, of course, is partly defined by fiat or even social consensus, although it can have an objectively physical component. The mixture of subjective and objective components in definitions of professional malpractice or failure has some bearing on the codes adopted by professionals for dealing with the uncertainty and ignorance entailed in practice. It is beyond our scope to discuss all professions' methods for coping with uncertainty, so I shall try to provide a 'maximal contrast' pair of professions on the basis of the subjective/objective dichotomy.

At the risk of provoking howls of outrage from several quarters, let us take structural design in civil engineering and jurisprudence as our contrast pair. Not all engineering entails objectively discernible failure or malpractice, but structural failure (e.g., the collapse of a building or bridge) comes fairly close. On the other hand, matters of justice are inherently socially created and often not tied to physical outcomes or consequences.

First let us consider what these two professional spheres have in common. One shared feature is an overriding conservativism towards speculation and trial-and-error methods of inquiry. Unlike the scientist (or at least a Popperian), neither the judge nor the structural engineer wish their operational rules and procedures to be falsified. They are interested in avoiding failure, and maximizing certainty or knowledge is merely one strategy among many for accomplishing this goal. Accordingly, they also share what might be called a tendency to err on the side of caution. In structural mechanics, this may entail constructions that are deliberately made much stronger than theory says they need be; in the lawcourts this proclivity is embodied in standards of proof and the injunction to presume innocence until guilt is proven. Thus structural designers and judges have similar goals and cautionary tendencies. Their strategies for avoiding failure, however, are markedly different.

Structural engineers usually encounter two kinds of design problems: One-off and mass-produced products. Usually, a product that is to be mass-produced usually is designed and constructed initially in prototype form, and the prototypes are then subjected to destructive testing (or 'proving'). Standards of proof for the prototypes usually are set at some level well above the wear or stress expected during normal service. One-off designs, on the other hand, usually cannot be tested in prototype form either because they are genuinely unique or such testing would be uneconomical. Instead, the engineer may destructively test prototype components, nondestructively test the entire structure (rarely done), or make theoretical calculations of maximal stresses and multiply those by safety factors to provide the design criteria.

The difference between these two design problems leads to quite distinct orientations towards ignorance on the part of the engineer. As Blockley (1980: 23) points out, in a one-off job no theory or prior information about similar structures is strictly applicable, and while the mass production designer can resolve considerable uncertainty during the prototype testing phase, the structural engineer cannot. Structural engineers therefore tend to be more conservative and less empirical than mass production engineers, and often rely more on old-style rules of thumb.

Traditional engineering criteria and rules for ensuring structural safety in the absence of empirical tests are based on two related strategies. The first is to estimate a 'worst case' load (or stress) scenario and apply it to theoretical computations of the capacity of the proposed structure, and the second is to use safety factors to ensure that the 'worst case' calculations exceed the likely actual loads and stresses by an appropriate margin of safety. Both strategies have been employed for some time. Straub (1949) recounts the procedures used in assessing damage to the dome of St. Peter's in Rome and the repairs required in 1742, in which stress and load calculations were made by a team of mathematicians and then adjusted using a safety factor of 2.

The precise nature of the models used to estimate worst-case loads or stresses and the magnitude of the safety factors employed are mutually intertwined. The simplest and oldest method for estimating stress uses elastic theory and arrives at a 'critical stress' estimate, which in turn is multiplied by a single safety factor to arrive at a 'permissible stress' criterion. Elastic theory provides rather liberal estimates of how much loading a structure can withstand, however, and its efficacy therefore depends on conservatively large safety factors being employed. The resulting structures tend to be overbuilt and expensive, but reducing the safety factors is clearly a risky undertaking.

A similar method is called the 'load factor method', and involves the use of two estimates of load: a 'working' load of the magnitude that could reasonably be expected during service, and a 'collapse' load which is the load under which the structure fails. The safety factor may then be calculated by dividing the collapse load by the working load. It is then up to arbitrators of standards to decide what an acceptable safety factor is. Like the permissible stress approach, this method's success hinges on the conservativism of the theoretical model that is used to calculate the collapse load, since that estimate often is not empirically based for one-off design problems. The most common conventional method of assessing how much tolerance for error a given theoretical estimate contains is to systematically vary the theoretical assumptions or relevant parameters, thereby revealing how sensitive the final estimates are to those variations. The engineer may then use this sensitivity analysis to provide lower and upper bounds on his or her estimates, or alternatively to demonstrate that the variability is too small to worry about.

More recent methods (such as limit state design as outlined in Blockley 1980: Ch. 4) are essentially combinations and generalizations of the permissible stress and load factor methods--- a sophisticated armamentarium of partial and componential safety factors along with complex methods for estimating stress and load-bearing capacities. One important addition has been made to these methods, however, and that is the overt use of both subjective and empirically based probability estimates of load magnitudes. A probability distribution of loads is usually generated either empirically or theoretically, and this is used to arrive at a design value that has a very low (even if unknown) probability of being exceeded.

In summary, engineers and others involved in construction arrive at a quantified consensus on safety margins and methods of estimating load-bearing capacities and stresses. The main counterbalancing consideration in construction is cost, hence the motivation to find more sophisticated structural theories that will permit smaller safety factors. Only very recently have they overtly used probability, but even when it is utterly notional or subjective, engineers seem relatively comfortable with quantitative guesses. The nature of that consensus differs across nations and even among engineering subfields, but the overwhelming emphasis is on attempting to measure 'how near to truth' they are rather than what truth itself is (Blockley 1980: 51).

Judges, like structural engineers, must operate most of the time with less than perfect information, witnesses who are unreliable, and 'facts' of doubtful standing. Often they have at best partial knowledge of how the information presented to them was obtained. Unlike the engineers, however, the judges may also have to contend with dishonesty on the part of witnesses and/or lawyers, as well as legitimate attempts by the disputants to manipulate the judge's perceptions or feelings. Furthermore, ultimately the judicial system does not either stand or fall, as does a physical structure, on the accuracy with which experts have plumbed natural laws, but instead on social and political criteria. These criteria are changeable, and the law itself is vulnerable to multiple interpretations and revisions.

Every judicial decision is guided by a standard of proof analogous to the engineer's testing criteria. In civil cases guilt must be decided 'on a balance of probabilities', while of course criminal cases require the tougher standard of proof 'beyond a reasonable doubt'. Unlike the engineering criteria and despite the probabilistic language, these phrases have seldom been made more specific. Eggleston (1978: 102), in his study of judicial probability, comments that judges appear to avoid explaining or elaborating on these traditional phrases for fear that their attempts to improve them will be found wanting by their peers. One is reminded of Nagel's (1961) anecdotal quotation of Lord Mansfield's advice to an appointed governor of a colony to decide judicial cases intuitively but to "...never give your reasons, for your judgement will probably be right, but your reasons will certainly be wrong."

The vagueness and long history of those verbal standards may also helpfully conceal a potential lack of consensus should their precise meanings to judges, jury, and litigants ever be communicated to one another. Where new verbal formulations (e.g., 'leaving only a remote possibility') have been tried, judges have been unable to agree on their meaning (Cohen and Christie 1970: 60). Nor does quantification of subjective probabilities help. Simon and Mahan (1971) asked mock jurors to provide numerical probability levels that corresponded in their opinions with the phrase 'beyond reasonable doubt' and obtained levels ranging from 0.7 to 0.9; but the judges from whom they requested the same kind of estimates required stricter levels. Furthermore, they found that by asking jurors to quantify their estimates of probable guilt, the percentage of attributions of guilt was lowered.

In addition to the standards of proof, many Western judiciary systems also impose a greater burden of proof on the prosecution than on the defense, in line with the adage that the defendant be assumed innocent until proven guilty. However, again the nature of this burden is unspecified for the most part, although there is a widely held craft-knowledge of what kinds of evidence will or will not convict, and lawyers and judges alike may refer to volumes of case precedents or even tomes on the subject of evidence itself.

As with standards of proof, judges and lawyers have avoided formalization or quantification of the 'weighing' of evidence; as Eggleston observes, the "legal profession as a whole has been notably suspicious of the learning of mathematicians and actuaries..."

(1978: 2-3). Suspicions notwithstanding, there have been many attempts to apply probability theory to legal problems. Indeed, Leibniz' early formulations of probability theory were motivated by problems of legal inference. Famous failures include probabilists of high rank: Laplace, Poisson, Cournot, Boole, and Keynes. To the extent that legal systems employ formal and quantitative rules of evidence, they greatly resemble the older engineering conventions. In fingerprinting, for example, experts in various Western countries require from 8 to more than 16 matching characteristics and no unexplained points of difference to risk the claim that two prints could have come from the same person (dare we call them rules of thumb?).

More often, rules of evidence are qualitative and broad. Traditional criteria for ascertaining the credibility of witness testimony include: (1) internal consistency of the account; (2) consistency and consensus with other credible witnesses; (3) evidence of mental soundness, competence, and credibility of the witness; (4) observable cues concerning witness mendacity, truthfulness, and confidence; and (5) the apparent (im)plausibility of the account. The first two criteria concern logical coherence and agreement, while the remaining three require considerable assumptions about human character and behavior. There is a large and rather inconsistent literature on how judges may ascertain whether perjury is being committed, and attempts to provide a 'scientific' basis for such inferences have provoked widespread controversy (e.g., the use of the polygraph in the U.S., U.K., and other countries. See the 1986 Report from the British Psychological Society).

Unlike the criteria for proof or evidence however, the legal systems in most Western nations are quite specific about the limits imposed on what kind of evidence may be admitted or heard, and for how long. These limits amount to (ir)relevancy rules which distinguish the matters that must be attended to by the judge and jury from those that may (or should) be ignored. A piece of evidence is considered relevant primarily if it may be used to prove the case. There are at least two ways in which this might happen: (1) If the evidence increases the credibility or believability of act(s) the court wishes to (dis)confirm; or (2) if the evidence is inconsistent with a relevant fact. Probativity is the dominant criterion for relevance insofar as it determines whether a piece of evidence has any bearing on the case, but veracity, credibility, adherence to proper procedures for obtaining evidence, and prejudiciality all define 'taboos' by which material may be banned from the court as inadmissible. For example, a defendant's prior criminal record, although verifiable and perhaps relevant, may be ruled inadmissible because it would prejudice the jury against the defendant.

The English system may be used as a model for explicating more specific points. Like most Western systems, it has two sets of rules pertaining to admissibility: Rules of exclusion and rules of extension. The former, of course, banish certain kinds of evidence from the proceedings. The latter permit the admission of evidence that does not appear to bear directly on the probativity of the case. Exclusion rules bar similar facts, hearsay, prejudicial material, privileged information, and evidence of character. Extension rules permit evidence of competence or credibility, surrounding detail or context, and facts 'deemed to be relevant' by the judge. Similar facts, hearsay, and rules of extension will be discussed in turn.

Similar facts refer to transactions or circumstances other than those which are the subject of the proceedings, and are inadmissible even if they might pertain to the case.

For instance, the question of whether the defendant was speeding on a particular road cannot be resolved by undertaking a survey and finding that 95% of the motorists traveling that road also exceed the speed limit; nor is evidence that the defendant has or has not exceeded speed limits on other occasions allowable. Major exceptions to this rule involve expert witness testimony and, under some conditions, evidence of the defendant's behavior under similar circumstances to those resulting in the trial.

Hearsay distinguishes secondhand reports and attributions from primary evidence. But the hearsay rule is so exclusionary that it creates problems. For example, if taken literally it would eliminate a considerable portion of eyewitness testimony. Consequently this rule has many exceptions, including confessions, admissions, statements in public documents, and statements contained in the 'res gestae' (the things done). A similar juggling of exceptions may be found in the rule excluding evidence of character. The obvious exception is the defamation case, but oddly witnesses are not protected by this rule, and quite often the accused is permitted to present evidence of good character.

The rules of extension may override the exclusionary rules, mainly at the discretion of the judge. Thus witnesses may be cross-examined for indications of their competence, qualifications, or credibility. Accounts of surrounding detail may be sought as a check on the internal consistency or reliability of witness testimony. And judges may deem relevant any fact that could be used as evidence for or against another probative fact.

What does the comparison between the strategies adopted by judges and structural engineers for coping with ignorance reveal? A crucial insight is that these two professions differ profoundly in how they manage the two main kinds of ignorance in our taxonomy (i.e., error and irrelevancy). More specifically, the engineers have explicit criteria for dealing with error while the judges are explicit on the matter of irrelevancy. The engineers' relevancy criteria are implicit while their framework for dealing with incompleteness and distortion is explicit and quantitative. Nor are those implicit relevancy criteria beyond questioning. After all, the conventional approaches to preventing structural failure under uncertainty ignore systemic uncertainty (as distinct from uncertainty about particular parameters and individual components), human errors, and consequences of structural failure. Any of these could be deemed relevant.

The judges, on the other hand, have explicit and fairly extensive relevancy criteria encompassing outright taboos, but implicit, qualitative, and somewhat privatized ways of dealing with incompleteness and distortion in evidence. The sources of ignorance in jurisprudence are more social and less grounded in apparently physical reality than in structural engineering, and the differences between their brands of normative pragmatism may be explicable by that alone. The explicit, public aspects of ignorance control in these two professions seem to correspond to those that are manageable and consensual.

By the same token, the irrelevancy rules employed by these pragmatists banish unmanageable or intractable varieties of ignorance. For judges, the salient problem is to limit and control the debate over evidence, and their primary resources for doing this are relevancy rules that form a part of 'due process'. Control over courtroom proceedings is essential if a judge is to preserve his or her professional standing. Engineers find it easier to agree on their subject matter if they tacitly eschew the psychological, social, or moral realms and restrict discourse to physical problems. For them, the salient problem is how to avoid physical evidence of structural failure which could threaten their professional reputations.

The foregoing discussion, then, illustrates a central thesis of this book: Not only is ignorance socially constructed, but so are the normative frameworks adopted by professionals or intellectuals for dealing with it. If we want to understand how and why mainstream orientations towards ignorance in several fields have changed during the last 20 years, then it will not suffice to restrict considerations to the philosophical or even the psychological. We will surely have to search for explanations that take social, political, and cultural factors into account as well.

2.3 The Ultimate Mental Health Project

I have hinted that modern trends in ignorance management are strategic responses to a widely perceived increase in the variety and extent of ignorance as well as a breakdown in the older consensus on absolute knowledge. I also have indicated that some fields have undergone a crisis that could be termed a 'loss of certainty', and their practitioners' coping strategies may tell us something about what is happening in other areas now. I believe this kind of crisis, and the strategies that evolved as a result, are nowhere more clearly visible than in modern mathematics. Accordingly, this Chapter ends with an account of developments during the crisis in pure mathematics. These developments centered around a challenge thrown down to his fellow mathematicians by David Hilbert, who described a series of problems whose solutions would provide absolute foundations for mathematical knowledge and inference.

The Hilbert Program in mathematics and its offshoots were direct responses to the blows against mathematical absolute certainty, and therefore full belief, that befell mathematicians in the 19th century. Extending Wittgenstein's metaphor slightly, some of the best mathematicians in the late 19th and early 20th centuries found themselves embarking on an ultimate 'mental health' project whose goal was to establish the criteria for mathematical sanity and thence to rehabilitate both the field and its practitioners. The difficulties that mathematicians encountered in pursuing complete certainty resemble, at least in surface features, many of the problems that have arisen in connection with ignorance in other intellectual fields. Although this chapter in mathematical history is well known, it remains remarkable that irreducible ignorance should have arisen and persisted in what many people regarded (and some still regard) as the knowledge paradigm least susceptible to chronic ignorance. Perhaps more than in any other field, mathematicians would seem to be well equipped to eliminate ignorance. There is some value, then, in employing hindsight to understand the strategies of ignorance management adopted in mathematics, the problems they raised, and the current situation.

First, however, we must appreciate some of the difficulties pursuing mathematicians in the 19th century. The truly halcyon days for mathematics arrived with the birth of Newtonian mechanics and a methodological shift in several of the physical sciences from emphasizing qualitative physical explanation to quantitative mathematical description and prediction. Galileo and Bacon were the precursors of this shift, heralded by Galileo's famous pronouncement in The Assayer of 1610 that the book of nature is written in mathematical language. While Newton, of course, was the main progenitor, the monumental contributions of such scholars as Euler, the Bernoullis, Laplace, Lagrange, Hamilton, and Jacobi not only raised the status of determinism to considerable heights but firmly installed mathematics as its terms of reference. By the end of the 18th century, many mathematicians and scientists had adopted the position later encapsulated by Kant's mildly tardy dictum that there is only as much real science as there is mathematics. Perhaps the most outstanding brief articulation of the full belief in the certitudes of mathematics during this time is the declaration by Laplace that

"An intellect which at any given moment knew all the forces that animate nature and the mutual positions of the beings that compose it...could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom: for such an intellect nothing could be uncertain..."

As Morris Kline (1980: 67) put it, natural law was mathematical law. Mathematical laws, therefore, were invested with universality and absolute certainty.

By the mid-19th century, however, several of the foremost mathematicians came to doubt that mathematical truths were either universal or absolute. As is well known, the seeds of this doubt germinated in the very mathematical fields held by many to be the most impermeable, namely geometry and algebra. The story of modern geometry is the more dramatic of the two, the prime mover being the pursuit of a justification of Euclid's so-called 'fifth postulate' (or 'parallel axiom'). The Jesuit priest Saccheri published an attempted proof by contradiction in 1733, in which he assumed a contrary postulate and claimed that it led to a contradiction in the resulting geometric system. However, by the mid 1700's at least a few mathematicians (e.g., Klugel, Lambert, and Kastner) went on record as recognizing that Saccheri's geometry was self-consistent and therefore non-Euclidean geometries were logically possible and constructible. Nonetheless, none of them pursued this realization to its conclusion. By 1813 Gauss had privately begun work on his non-Euclidean geometry which he apparently refrained from publishing for fear of the outcry it would raise. Lobachevsky and Bolyai, of course, received the majority of the credit for publicly articulating non-Euclidean geometries, and Riemann for the most famous case of a physically relevant nonstandard geometry.

Gauss reacted to the loss of absolutism in geometry by reposing his faith in arithmetic. Geometry, he claimed, was akin to mechanics, and its foundations ultimately were experiential. Arithmetic, on the other hand, was purely a priori and based on obviously certain numerical truths. Ironically, the foundations of algebra itself were threatened only a short time later by several developments which resembled those in geometry. Hamilton's quaternions were motivated by problems in physics, but their practical success came with a price: Quaternion multiplication is not commutative and thereby violates the 'absolute' properties of the algebra shared by all real and complex numbers. Other useful but noncommutative algebras soon followed (e.g., Cayley's matrix algebra and Grassmann's family of such algebras). Worse still, physicists such as Helmholtz pointed out that our most basic numerical and arithmetic concepts are grounded in experience and not a priori truths.

At about the same time as these unsettling developments were taking place, many excellent mathematicians were becoming uneasy about the lack of properly logical underpinnings for the calculus, particularly for the difficult concepts of limits and series which involved infinities and infinitesimals. The immense practical successes of the calculus promoted its application despite its unsound foundations, and those applications led increasingly both to exceptions to seemingly 'universal' theorems and to monsters in the form of bizarre logical paradoxes. By the latter half of the 19th century, many first-rank mathematicians were calling for an end to the confusion and chaotic developments that characterized the preceding 150 years. What strategies did they adopt for this return to sanity?

The classical strategies may be divided, roughly speaking, into three groups: Reductionism, banishment, and pragmatism. To use a metaphor that I shall extend later on, reductionism is essentially a rehabilitative or corrective impulse, while banishment is an attempt to bar problematic elements from mathematics. Reductionism is therefore an inclusionary form of control, while banishment is exclusionary. One of the most frequently used forms of banishment is an irrelevance rule, which dictates the boundaries of what 'proper' science or mathematics is and places unwanted uncertainties in the domain of irrelevant phenomena. Pragmatism, on the other hand, amounts to an abandonment of attempts to deal with the difficulties insofar as it entails evaluating mathematical concepts in terms of their utility for solving practical problems in other domains. Most sophisticated strategies for developing a general mathematical overview include elements of all three strategies. Nonetheless, some of them are identifiable by their emphasis on one or another such strategy.

Reductionists attempt to bring rigor to mathematics by reducing it to another system widely felt to be rigorous. This strategy has an impressive mathematical pedigree, and from ancient times manifested itself in various attempts to declare one mathematical field supreme over all the others. From the time of classical Greek civilization to the era of Descartes and Pascal, geometry held the crown. By the time Newton and his contemporaries had completed their work on the calculus, however, algebra increasingly gained favour, and by the Second International Congress of Mathematicians in 1900, Poincare could declare that mathematics had been arithmetized (Kline 1980: 182).

The logicists were the dominant reductionists in the modern crisis, claiming that any mathematical intuition was suspect until confirmed by analysis in terms of classical first-order (and sometimes second-order) logic. The most famous logicists included Frege, Whitehead, and Russell. Russell is the most widely quoted of the three, and probably the most accessible to the nonmathematician. For some twenty years he propounded logicism and defended it against its many critics. His recollection of his motivations in those days leaves no doubt of his agenda: "I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere... if certainty were indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure" (Russell 1958). Here, then, is the ultimate mental health project incarnate.

Russell and his colleagues argued not only that mathematics is ultimately reducible to logic, but that the two are coextensive. In his 1919 Introduction to Mathematical Philosophy, Russell claimed that no clearcut distinction could be drawn between logical and mathematical axioms, propositions, and methods of deduction. To fully appreciate how radical this thesis seemed to many at the time, we must realize that by the mid-19th century the notion of mathematical proof was so disreputable that some quite able mathematicians hardly bothered with proofs at all. To them, an appeal to logic as the cornerstone of mathematics was an irrelevancy. However, it is noteworthy that not all mathematicians understood the same thing by the term 'logic'. Until the mid-1800s, Aristotelian logic dominated both philosophical and mathematical discourse on logical matters. But the logicists' program was based on and enabled by the modern development of symbolic logic, initiated by Boole and De Morgan, and further developed by Peirce.

This new brand of logic, in fact, was derived in good part from ideas in algebra and therefore could be said to be mathematized logic.

Not all of the logicists' strategies were reductionistic, however. Even Whitehead and Russell found it necessary and sometimes convenient to use banishment tactics. The most famous of these is the Theory of Types, in which individual objects belong to a type 0 level, sets of objects belong to type 1, sets of sets belong to type 2, etc. Propositions that cross the boundaries are not allowed, so it becomes impermissible to speak of a set that belongs to itself.

Banishment is an even older strategy than reductionism, with roots in ancient mathematics. The primary objective is to find a way of avoiding paradoxes or monsters by constructing definitions and criteria that bar them in principle. Diophantus and his contemporary Arithmeticians declared negative numbers 'impossible' and 'absurd' (Newman 1956, V.1: 115), pointing out the apparent incomprehensibility of subtracting a larger quantity from a smaller quantity. These numbers were barred by the Arithmeticians from proper mathematical inquiry. Similar reactions may be observed as mathematicians encountered irrational, imaginary, and complex numbers. Their very names evoke their disrepute.

The modern banishers are the intuitionists, who directly oppose reductionism of any kind, and particularly logicism. The initiators of intuitionism included Baire, Borel, Kronecker, Lebesgue, and Poincare, all of whom raised objections to the axiomatics of the late 19th century and the logicists' programs in the early 20th. The fully-fledged intuitionist perspective, however, was expounded by Brouwer and elaborated by Weyl. Brouwer declared that mathematics is an autonomous mental activity, divorced from the world outside the mind, and independent of language. It is synthetic rather than derivative, and therefore cannot be based on some other system of thought or language such as logic. Intuition determines the validity of fundamental ideas, not logic. Certain mathematical ideas are intuitively clear but others are not and therefore cannot be accepted as part of true mathematics, whatever their historical roles or logical status.

It is at this point that intuitionism reveals itself as a banishment strategy. Thus, Brouwer claimed that natural numbers, addition, multiplication, and mathematical induction are intuitively sound. The latter permits infinite sets, but only those that are 'potentially' and constructibly infinite. Cantor's transfinites, Zermelo's axiom of choice, the continuum hypothesis, and other 'nonintuitive' consequences of logical or formal manipulations are banned. Moreover, logic is held by intuitionists to belong to language, not to mathematical thought. Not all principles in logic are acceptable to intuition, and so the existence of paradoxes indicates an undisciplined or improper application of logic, not a fault in mathematics. Brouwer and Weyl both castigated logicians for abstracting the Law of the Excluded Middle beyond its initial intuitively justifiable application to finite sets, and thereupon banished the Law of the Excluded Middle from arguments involving infinities. Partly as a consequence of this, the intuitionists also disallowed most reductio ad absurdum proofs (proof by contradiction) and many nonconstructive existence proofs (which establish the existence of a mathematical entity without specifying how to construct or observe it). A number of famous and widely used theorems were disqualified by the intuitionists.

Neither the reductionist or the banishment strategies provided satisfactory solutions to the problems raised in the 19th century. Logicism was widely attacked, both for specific axioms or arguments that opponents founds questionable, and more generally by those who rejected either its somewhat Kantian a priorism or the primacy of logic over mathematics. Though criticised on various specific counts, logicism's chief shortcoming is its inability to guarantee security against not only the paradoxes and monsters it was designed to handle, but any unforseen difficulties of the same kind.

The major complaints against the intuitionist school of thought probably were identical to those facing the Diophantines, namely that they were throwing the baby out with the bathwater. Intuitionism, if taken seriously, dismantled so much classical and modern mathematics that the field would be devastatingly impoverished. Although the intuitionists themselves have devoted considerable effort to reconstructing mathematics in accordance with their philosophy, both the slow pace and cumbersome complexity of that reconstruction have earned them many criticisms among practicing mathematicians. Hilbert spoke for many of them in his well-known 1927 declaration: "For, compared with the immense expanse of modern mathematics, what would the wretched remnants mean, the few isolated results incomplete and unrelated, that the intuitionists have obtained."

Furthermore, there is the awkward fact that the intuitionists themselves do not always agree on what is intuitive or not and how to establish this. In recent times the "father of fractals", Mandelbrot (1983), has argued that many so-called "monstrous" mathematical creations (such as the Peano curve that covers a square) are intuitively obvious and clearly manifested in natural forms, contrary to mainstream mathematical commentators such as Hahn (1956) and others before him. In fact, Mandelbrot's own position embodies the postmodern rejection of both classical strategies for handling mathematical creative uncertainty: "In any event, the typical mathematician's view of what is intuitive is wholly unreliable; it is impossible to permit it to serve as a guide in model making; mathematics is too important to be abandoned to fanatic logicians" (1983: 150).

Some modern classicists have attempted to combine the banishment and reductionist strategies. One such approach, which still enjoys some popularity among mathematicians, was constructed by Zermelo, Fraenkel, and their fellow workers in set theory. The Zermelo-Fraenkel revision of 'naive' set theory amounts to a compromise between the reductionist and banishment strategies, which may partly explain its appeal. Another appealing aspect is that some of the most galling monsters and paradoxes were initially generated by Cantor's version of set theory, which itself was an attempt to provide a foundation for the calculus. The Zermelo-Fraenkel axiomatization of set theory avoids the all-inclusive set and hence the paradoxes associated with that concept, but retains a sufficiently rich set theory (e.g., permitting transfinite cardinals and certain other concepts that go beyond experience and/or intuition) to satisfy the requirements of classical analysis.

The main criticisms raised against the Zermelo-Fraenkel scheme were technical, and concerned specific axioms. The axiom of choice, for instance, created considerable controversy but most of the initial arguments questioned its technical feasibility or intuitive plausibility rather than its logical or formal potential for generating paradoxes and monsters. Eventually the controversy was settled in a manner of speaking by proofs that the axiom of choice is both consistent relative to Cantorian set theory (Godel) and independent of the other axioms (P. Cohen). These proofs ensured that the axiom of choice could be added to or dropped from the Cantorian collection of axioms without damaging the formal system (Kramer 1970: 595), thereby rendering its inclusion in set theory a matter of choice indeed. The price for this is, however, that the axiom of choice itself is not decidably true or false within the Zermelo-Fraenkel system, and mathematicians are therefore bereft of any clearcut reasons for choosing to include it or not.

Reductionism and banishment are examples of traditional Western ways of dealing with ignorance, namely by outright reduction or elimination. Furthermore, they are not managerial strategies, insofar as neither of them approach the problem of ignorance in an anticipatory or generalized fashion. Instead they are crisis-oriented attempts to solve here-and-now problems posed by specific paradoxes and mathematical monsters. A more modern managerial approach had to await an attack on foundational problems from David Hilbert.

Hilbert not only was the first mathematician to coherently delineate several of the major problems facing the foundations of mathematics, but he also dismissed both classical logicism and intuitionism and proposed his own strategy, as he put it, "to establish once and for all the certitude of mathematical methods" (a quotation from his 1925 paper "On the Infinite", as cited in Kline 1980: 246). Hilbert's approach is often referred to as the "formalist" school of mathematical thought. He began with the assertion that a proper mathematical science must include axioms and concepts of both mathematics and logic, and all mathematical or logical statements must be expressed in a symbolic form that frees them of all ambiguity or vagueness. The symbols themselves need not have any meaning or intuitive sensibility. They must, however, be manipulated in a logically consistent fashion.

The kind of certainty sought by Hilbert differed in one important respect from that of the logicians and intuitionists, in that he wished to construct proofs of absolute consistency for mathematical systems. Indeed, he had already shown that Euclidean geometry is consistent if arithmetic is consistent. A proof of the consistency of arithmetic became the formalists' Holy Grail during the 1920's. The second goal of Hilbert and the formalists was an absolute proof of 'completeness'. A mathematical theory is consistent if it never generates contradictions, but it is complete if every possible meaningful statement generated by it is decidably true or false.

The crucial difference between this strategy and its classical forebears lies in the level of analysis involved. The classical approaches to establishing certainty operate at the first-order level of mathematics or logic itself. Hilbert's program required a second-order approach, that is, a meta-mathematical methodology. Logicism could not guarantee that new paradoxes or undecidable propositions would not arise someday in the future, nor could intuitionism guarantee that no unforeseen monsters would pop up inside the enclosure they had built. If Hilbert's program could be achieved, however, then the unknown in mathematics would literally be tamed.

The ultimate mental health program ended when Godel published his famous 1931 paper. The two most devastating results dashed the hopes Hilbert and his followers had of establishing absolute certainty in mathematics on terms acceptable to mathematicians of the time. First, Godel demonstrated that the consistency of any mathematical system as extensive as whole-number arithmetic cannot be proved by the logical principles adopted in any of the foundational schools of thought. Secondly, and more fundamentally, he established that if any formal system that encompasses whole-number theory is consistent, it must be incomplete. This result applies to both first-order and second-order predicate logic. Although some of the undecidable propositions generated by a self-consistent arithmetic could be decidable by arguments using logical principles that transcend that arithmetic, the transcendent system itself might not be consistent.

The problem of incompleteness was driven home in an even more profound form by the Lowenheim-Skolem results, which implied that any axiomatic system is vulnerable to unintended interpretations or models. One of the primary aims of axiomatic methods is to fully describe and define a particular 'species' of mathematical entities (e.g., the positive whole numbers). Lowenheim and Skolem's theorems proved that for any system of axioms there are other unanticipated 'species' that fulfill the axioms and yet were not intended to be included in the original class encompassed by the axioms themselves. A major source of these new mathematical monsters is the inevitable inclusion in any axiomatic system of some undefined terms. The upshot of these results is that mathematical reality cannot be described unambiguously by axiomatic methods.

In short, ignorance in mathematics is here to stay. Mathematicians now labour under what appears to be unavoidable twin spectres of doom: The Scylla of inconsistency and the Charybdis of incompleteness. Postmodern mathematicians have adopted various identifiable strategies for living with irreducible ignorance, and these echo recent trends in Western intellectual culture at large while providing particularly instructive examples of those trends.

One of the most noticeable trends is the relativization and consequent privatization of mathematical truth (cf. Wilder 1981: 39-41), a phenomenon remarkably similar to the modern privatization of religion. This tendency has been aided by the rapidly escalating specialization among mathematicians since the Second World War. The result is a subdivision and resubdivision of territory which, for want of a better term, I shall call "suburbanization". Specialists form homogeneous epistemic subcommunities within suburbs and either outright ignore or agree to disagree about the nature of mathematical truth with mathematicians from other suburbs. These practices amount to more than mere specialization, since they also involve the erection of boundaries.

Suburbanization not only involves dividing up mathematical subject matter and domains, but also selecting specific approaches to fundamentals. Thus today we find more than a dozen major versions of set theory, encompassing more than 50 axioms of various kinds. And with suburbanization has come the mathematical equivalent of urban planners, imperialists of a different sort than the would-be hegemonists of yesteryear. These system-builders engage in meta-axiomatics, which amounts to specifying which axioms may be added to or dropped from various versions of set theory without damaging the system. Having determined the menu, the choice of axioms is then left to the user.

A fairly typical example of this brand of theorizing is Vopenka's (1979) "alternative set theory" (AST). As one proponent of AST puts it, "...even the property 'to be a book' is not quite precisely defined... the decision whether a collection of objects... will be formalized as a set or as a proper semiset depends on our standpoint" (Sochor 1984: 174). Sochor then exemplifies the grander scheme of mathematical suburbia according to AST: "...Cantor's set theory becomes a study of one particular class of a certain type of extended universe studied by AST" (Sochor 1984: 186).

What criteria are used by the suburbanites for selecting axioms, guiding creative activity, and deciding the legitimacy of truth-claims? The postmodern choices of criteria include, of course, the classical schools amongst the options. But for the nonclassicists, the determining criteria tend to be either utility or aesthetics. If we cannot attain absolute certainty, then at least mathematics should be useful in applications (the 'applied' point of view) or beautiful to behold (the 'pure' viewpoint). To absolutists, these criteria beg the question. For instance, even granting the much remarked and possibly arbitrary success mathematics has had in modeling the physical universe, mathematicians (and other scientists) still are unable to answer the question of why that success should be the case and whether it will continue. Heaviside, on the other hand, spoke for most applied mathematicians and scientists when he asked whether he should refuse his dinner because he did not understand the digestive process.

The most important strategy for handling irreducible ignorance in mathematics to emerge in recent times, however, is to apply mathematical methods to the study of ignorance itself. Wilder (1981: 82) has suggested that pure mathematicians are less threatened by paradox or undeciability in formal systems than their predecessors, which if true is a reflection of a more managerial approach to mathematical ignorance. Thus, the specific nature of paradoxes, the types of mathematical propositions that are undecidable within particular systems, and the formalization and quantitative measurement of vagueness, ambiguity, and complexity in mathematics have all become topics of study in their own right (e.g., Skala et al. 1984).

These developments are, like Hilbert's program, a 'second-order' strategy for dealing with uncertainty. Unlike Hilbert's attempt to eliminate ignorance completely, however, this new second-order strategy primarily attempts to impose some order on ignorance by understanding and adapting to it. Therein lies its importance, as an exemplar of a major shift in Western intellectual culture away from the pursuit of absolute certainty. I shall not attempt to explain these developments in the recent history of mathematics here, since their primary utility at this point is illustration and suggestion. The sequential unfolding of strategies by mathematicians for dealing with ignorance suggests a crude but plausible model that will serve as a template later on.

The sequence of strategies may be briefly summarized in four stages. First outright banishment is tried, by way of directly eliminating uncertainty and clearly demarcating the boundary between the true and the false. If banishment eliminates some useful techniques or concepts, however, then pragmatists usually argue for inclusionary modes of control. The second stage therefore involves attempts to rehabilitate the 'monsters', and the most popular strategy is reductionism.

When reductionist or other inclusionary efforts generate their own monsters, a third stage begins with a leap to second-order strategies for taming first-order ignorance and uncertainty. For the mathematicians, this strategy entailed the search for proofs of absolute consistency and completeness. More generally, such strategies involve the first serious attempts to describe, classify, and explain ignorance itself. The failure of the second-order strategies leads to the fourth stage, which continues the study of ignorance and uncertainty but also entails suburbanization, relativistic coexistence or epistemic pluralism, and the abandonment of the pursuit of certainty.

The explanation of this history poses a fascinating problem in the sociology of knowledge (see Chapter 6). As the material in section 2.2 demonstrated, and has sociologists of science have argued for some years, what passes for knowledge and ignorance in mathematics is socially constructed by the community of mathematicians.

That construction cannot be explained entirely by referring to the strategies mathematicians have employed to deal with ignorance. Fuller explanations must await an investigation into what accounts the human sciences offer of how people deal with ignorance. That investigation will be taken up in the second part of this book.

Meanwhile, this section has provided a stage-model of strategies for coping with ignorance. I will employ this model in Chapters 3 and 4 to provide an account of the developments stemming from probability theory and its offshoots. We will find that the probabilists have used the classical first-order strategies of banishment and reduction in their attempts to tame uncertainty. The 'new look' in the mathematics of uncertainty, with its nonprobabilistic formalisms and appeals to nonstandard logics, represents the leap into second-order strategies and the prospect of irreducible uncertainty.